



COMMON PRE-BOARD EXAMINATION: 2022-23



Class-XII Subject: MATHEMATICS (041)

Date: 12/01/2023

Time: 3 hours

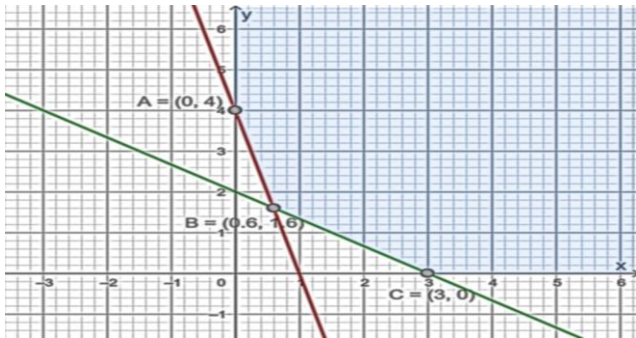
M.M.: 80

General Instructions :


1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-a type question of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-a type question of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-a type question of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (Multiple Choice Questions) Each question carries 1 mark

1. If $\tan^{-1}x = \pi/3$ for some $x \in \mathbb{R}$, then the value of $\cot^{-1}x$ is
(a) $\pi/3$ (b) $\pi/6$ (c) $-\pi/3$ (d) $-\pi/6$
2. The maximum number of equivalence relations on the set $A = \{a, b, c\}$ are
(a) 1 (b) 2 (c) 3 (d) 5
3. If A is a square matrix of order 3, $|A'| = -4$, then $|AA'| =$
(a) 16 (b) -16 (c) 4 (d) -4
4. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^2 equals to
(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
5. The matrix product $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} (4 \ 5 \ 2) \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ equals to:
(a) $\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ 9 \\ 6 \end{pmatrix}$ (c) $\begin{pmatrix} 3 \\ 6 \\ -9 \end{pmatrix}$ (d) None of these
6. The value of $|\text{adj } A|$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is:
(a) -2 (b) 1 (c) -1 (d) 2
7. The value of k if $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & 2 \end{vmatrix} = -33$, is:
(a) -1 (b) $\frac{5}{7}$ (c) $\frac{22}{7}$ (d) $\frac{33}{7}$
8. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & , \text{ if } x \neq 0 \\ k & , \text{ if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

	a) 0 b) 2 c) 1 d) 1.5
9.	If $y = \tan^{-1} \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)$, then $\frac{dy}{dx}$ is : (a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
10.	The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^{17} x \, dx$ (a) 1 (b) 0 (c) -1 (d) π
11.	The value of $\int \sec^2(1-x) dx$ (a) $-\tan(1-x) + C$ (b) $\tan(1-x) + C$ (c) $-\sec(1-x) + C$ (d) <i>None of these</i>
12.	The integrating factor for the given differential equation is $(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$ (a) $\log(1+x^2)$ (b) $1+x^2$ (c) $2x$ (d) $\tan^{-1}(x)$
13.	The order of differential equation $\frac{d^4 y}{dx^4} + \sin\left(\frac{d^2 y}{dx^2}\right) = 0$ is : (a) 2 (b) 4 (c) 6 (d) not defined
14.	If $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$, then the value of $\vec{a} \cdot \vec{b}$ is: (a) 3 (b) -3 (c) 4 (d) None of these
15.	The direction cosines of the line joining (2, -4, 5) and (0, -6, 4) are: (a) 2, 2, 1 (b) -2, -2, -1 (c) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (d) None of these
16.	If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then (a) $A \subseteq B$ (b) $B \subseteq A$ (c) $B \neq \phi$ (d) $A = \phi$
17.	The corner points of the shaded unbounded feasible region of an LPP are A(0, 4), B(0.6, 1.6) and C(3, 0) as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at (a) (0.6, 1.6) only (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only (d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
	
18.	Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A' \cap B')$ is (a) 0.9 (b) 0.18 (c) 0.28 (d) 0.1
<p style="text-align: center;">ASSERTION-REASON BASED QUESTIONS</p> <p>In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.</p> <p>(a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A.</p>	

	(c) A is true but R is false. (d) A is false but R is true.																		
19.	Assertion (A):-If A and B are symmetric matrices of same order then $AB-BA$ is also a symmetric matrix., Reason(R):Any square matrix A is said to be skew symmetric matrix if $A = -A'$, where A' is the transpose of matrix A.																		
20.	Assertion (A): $f(x)=e^x$ is an increasing function, $\forall x \in R$ Reason (R): If $f'(x) \leq 0$, then $f(x)$ is an increasing function.																		
	SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each																		
21.	a) Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$. b) Find the value of $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$. OR Let $A=R-\{3\}$ and $B=R-\{1\}$ $f:A \rightarrow B$ be defined as $f(x)=\frac{x-2}{x-3} \forall x \in A$, Then show that f is bijective																		
22.	An edge of a variable cube is increasing at the rate of 3cm/sec. How is the volume of the cube increasing when the edge is 10cm long?																		
23.	Show that $\left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right)=2\left(\vec{a} \times \vec{b}\right)$.																		
24.	Find λ when scalar projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units OR Find the vector and Cartesian equations of the line through the point (5, 2, 4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.																		
25.	Find $\frac{dy}{dx}$, if $x = a (1 - \cos \theta)$ and $y = a (\theta - \sin \theta)$.																		
	SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)																		
26.	Evaluate: $\int \frac{1}{\sqrt{8+3x-x^2}} dx$																		
27.	The probability of two students A and B coming to school on time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events (A coming on time and B coming on time are independent), find the probability of only one of them coming to school on time. OR A random variable has following probability distribution: <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(X)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr></table> Determine: (i) k (ii) $P(X < 3)$ (ii) $P(X > 6)$	X	0	1	2	3	4	5	6	7	P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$
X	0	1	2	3	4	5	6	7											
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$											
28.	Evaluate: $\int \frac{x}{(x-1)^2(x+2)} dx$																		
29.	Solve the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)dx$ OR Solve the differential equation $xcos\left(\frac{y}{x}\right)\frac{dy}{dx} = ycos\left(\frac{y}{x}\right) + x$.																		

30.	<p>Solve the following Linear Programming Problem graphically:</p> <p>Maximise $Z = 3x + 9y$ subject to $x + 3y \leq 60$; $x + y \geq 10$; $x \leq y$; $x \geq 0, y \geq 0$</p>
31.	<p>Evaluate $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.</p> <p style="text-align: center;">OR</p> <p>Evaluate $I = \int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$.</p>
	<p>SECTION D</p> <p>(This section comprises of long answer-type questions (LA) of 5 marks each)</p>
32.	<p>Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the straight line $\frac{x}{3} + \frac{y}{2} = 1$</p>
33.	<p>Define the relation R in the set $N \times N$ as follows: For $(a, b), (c, d) \in N \times N$, $(a, b) R (c, d)$ iff $ad = bc$. Prove that R is an equivalence relation in $N \times N$.</p> <p style="text-align: center;">OR</p> <p>For real numbers x and y, we write xRy, implying that $x - y + \sqrt{2}$ is an irrational number. Then, check relation R is reflexive, symmetric, or transitive?</p>
34.	<p>Find the foot of perpendicular from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ Also, find the perpendicular distance from the point to the line.</p> <p style="text-align: center;">OR</p> <p>Find the shortest distance between the lines</p> $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ $\vec{r} = (1 + s)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$
35.	<p>If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ find A^{-1}.</p> <p>Hence solve the equation $x + 2y - 3z = -4$; $2x + 3y + 2z = 2$; $3x - 3y - 4z = 11$.</p>
	<p>SECTION E</p> <p>(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)</p>
36.	<p>Case-Study 1: Read the following passage and answer the questions given below.</p> <div style="text-align: center;">  </div> <p>The temperature of a person during an intestinal illness is given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$, $-3 \leq x \leq 3$, where $f(x)$ is the temperature in $^{\circ}\text{F}$ at x days.</p> <ol style="list-style-type: none"> Is the function differentiable in the interval $(-3, 3)$? Justify your answer. Find the critical points. Find the intervals in which the function is strictly increasing,

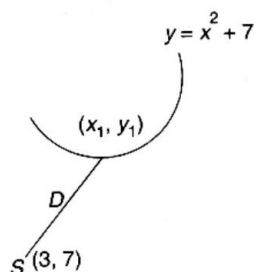
OR

(iii) Find the absolute maximum/absolute minimum values of the function in the interval $[-3, 3]$.

37.

Case-Study 2: Read the following passage and answer the questions given below.

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at $(3, 7)$ want to shoot down the helicopter when it is nearest to him.



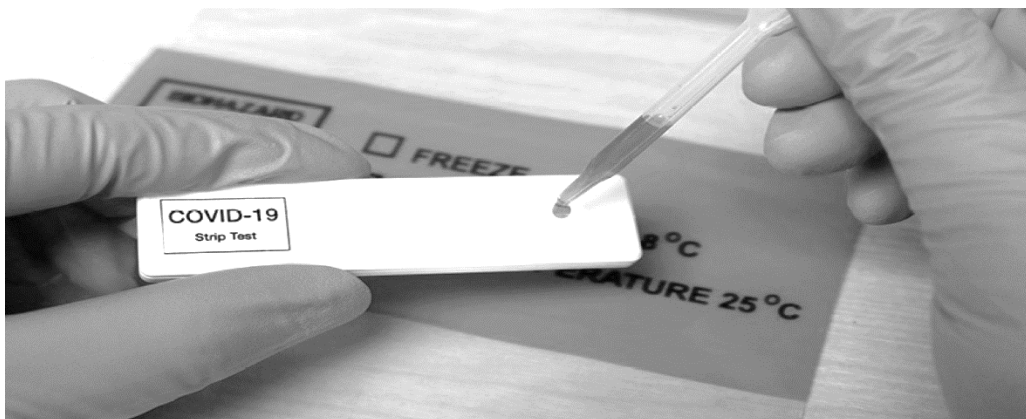
- (i) If $P(x_1, y_1)$ be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at $(3, 7)$.
- (ii) Find the critical point such that distance is minimum.
- (iii) Verify by second derivative test that distance is minimum at $(1, 8)$.

OR

(iii) Find the minimum distance between soldier and helicopter?

38.

Case-Study 3 Read the following passage and answer the questions given below.



A Covid-19 test is 99% effective in detecting Coronavirus disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e., if a healthy person is tested, then, with probability 0.005, the test will imply he has the Coronavirus disease). If 0.1 percent of the population actually has the Coronavirus disease.

- (i) What is the total probability that a person selected from this population will be tested positive?
- (ii) What is the probability that a person has the Coronavirus disease given that his test result is positive?